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ON PROPERTY I FOR KNOTS IN S^3

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ABSTRACT. This paper deals with the question of which knot surgeries on S^3 can yield 3-manifolds homeomorphic to, or with the same fundamental group as, the Poincaré homology 3-sphere.

1. INTRODUCTION

Problem 3.6(D) in [19] asks whether there is a 3-sphere which can be obtained by surgery on an infinite number of distinct knots in S^3. Examples of homology 3-spheres which can be obtained by surgery on two or finitely many distinct knots in S^3 have been found. However, the Poincaré homology 3-sphere, Kirby made a remark after Problem 3.6(D), seems only obtainable from 1-surgery on the right-hand trefoil knot (or, reversing orientation, from -1-surgery on the left-hand trefoil knot). This paper is devoted to provide evidence to support this observation.

The Poincaré homology 3-sphere, first constructed by Poincaré, is a very special manifold. It seems to be the first known example of a nonsimply connected closed 3-manifold with trivial first homology group. It has several interesting descriptions (see [27, 20]). Here we only mention three of them which are relevant to this paper:

(i) the manifold obtained by 1-surgery on the right-hand trefoil knot;

(ii) the quotient space of S^3 under a free action of the binary icosahedral group $I_{120} = \{x, y; x^2 = (xy)^3 = y^5, x^4 = 1\}$;

(iii) the 2-fold (3-fold, 5-fold) cyclic branched cover of S^3 branched over the $(3, 5)$ $((2, 5), (2, 3))$ torus knot.

By (ii) the fundamental group of the Poincaré homology 3-sphere is the binary icosahedral group I_{120}. This group has order 120 and trivial abelianization. So far it is not known if the Poincaré homology 3-sphere is the only homology 3-sphere with nontrivial finite fundamental group.

Definition. A knot K in S^3 has property I if every surgery along K does not yield a manifold M with $\pi_1(M) = I_{120}$. A knot K in S^3 has *property* $\hat{I}$ if every surgery along K does not yield the Poincaré homology 3-sphere.

Of course the trefoil knot does not satisfy property $\hat{\mathrm{I}}$.

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Conjecture I $(\hat{\mathrm{I}})$. Every nontrefoil knot in S^3 has property I $(\hat{\mathrm{I}})$.

Recall that property P $(\hat{\mathrm{P}})$ conjecture states that every nontrivial surgery along a nontrivial knot in S^3 does not yield a homotopy 3-sphere (the 3-sphere). The property $\hat{\mathrm{P}}$ conjecture has been proved recently by Gordon and Luecke [12]. It is known that if the fundamental group of a homology 3-sphere is finite then it is either the trivial group or else the group I_{120} [18]. Therefore property I and property P together are equivalent to property PI defined as follows.

Definition. A knot K in S^3 has property PI if every homology 3-sphere obtained by a nontrivial surgery along K has infinite fundamental group.

Conjecture PI. Every nontrivial nontrefoil knot in S^3 has property PI.

Much research has been carried out to prove property P. No literature, however, has been found dealing specifically with the generalized problem we just raised above. As we will see, property P and property I $(\hat{\mathrm{I}})$ have certain connections and common features; some techniques which work for property P can also be generalized to work for property I $(\hat{\mathrm{I}})$. However in general the two properties do not imply each other. Certain knots (e.g., slice knots) are found to have property $\hat{\mathrm{I}}$ but are not known whether or not to have property P. In many cases property I seems a harder problem.

The rest of the paper is organized as follows. Section 2 contains a complete classification of cyclic group actions on the Poincaré homology 3-sphere with 1-dimensional fixed point sets. The main purpose of §2 is to obtain Corollary 2.3 which plays a role for property I problem similarly as the Smith conjecture does for property P problem. In §3 several popular classes of knots in S^3 are proved to have property I or $\hat{\mathrm{I}}$. Various techniques and results in 3-manifold theory and knot theory are applied. The paper concludes in §4 with further remarks and open questions.

We work throughout in the PL category and we refer to [15 and 27] for basic terminology.

This paper is taken from part of the author's Ph.D. thesis [32] and as such the author is indebted a lot to his supervisor Erhard Luft. The author thanks the referee for his (her) comments, in particular for pointing out a gap in the earlier version of this paper.

2. Cyclic actions on the Poincaré homology 3-sphere

In this section we give a complete description of orientation preserving isometric cyclic actions on the Poincaré homology 3-sphere, denoted by D^3 (Theorem 2.2). Combining Theorem 2.2 with a result of Thurston, we obtain a classification of cyclic actions on D^3 with fixed point sets of dimension 1 (Corollary 2.3) which will be applied in §3.

The following lemma will be used in the proof of Theorem 2.2. Its proof is elementary and is thus omitted here.

Lemma 2.1. *Let X be a path connected, locally path connected and semilocally simply connected space, and let $p\colon \tilde{X} \to X$ be a universal covering projection. Let G be a group of homeomorphisms of X and let Γ be the group of covering transformations. Define $\tilde{G} = \{\tilde{g}\,;\, \tilde{g}\colon \tilde{X} \to \tilde{X}$ a map with $p\tilde{g} = gp$ for some*

$g \in G\}$. *Then*

1. $\widetilde{G}$ *is a group of homeomorphisms of* $\widetilde{X}$.

2. *If* $N \subset G$ *is a normal subgroup, then* $\widetilde{N} \subset \widetilde{G}$ *is a normal subgroup. In particular,* $\Gamma = \widetilde{\{1\}} \subset \widetilde{G}$ *is a normal subgroup.*

3. *For each* $\tilde{g} \in \widetilde{G}$ *the element* $g \in G$ *with* $p\tilde{g} = gp$ *is unique, the map* $p_*: \widetilde{G} \to G$ *defined by* $p_*(\tilde{g}) = g$ *is an epimorphism, and the sequence* $1 \to \Gamma \to \widetilde{G} \xrightarrow{p_*} G \to 1$ *is exact.*

In Theorem 2.2 below, we present D^3 as 3-dimensional space form; i.e., consider the orthogonal action of $SO(4)$ on S^3 and let $D^3 = S^3/I_{120}$ where I_{120} is a subgroup of $SO(4)$. If $(I_{120})_1, (I_{120})_2 \subset SO(4)$ are subgroups isomorphic to I_{120}, it follows from [28, Theorems 4.10 and 4.11] that they are conjugate in $O(4)$. Consequently $S^3/(I_{120})_1$ and $S^3/(I_{120})_2$ are isometric. Thus $D^3 = S^3/I_{120}$ is independent of the choice of the subgroup $I_{120} \subset SO(4)$.

Theorem 2.2. (i) *For each integer* $n > 1$ *there is an orientation preserving isometric* Z_n *action on* D^3.

(ii) *Up to conjugation by an isometry, such a* Z_n *action is unique for each* n.

(iii) *If* n *is relative prime to* 2, 3, *and* 5, *then the* Z_n *action is free; if* n *is not prime to* 2, 3, *or* 5, *then exactly those elements of* Z_n *which have orders* 2, 3, *or* 5 *have fixed point sets and the fixed point set of each such element is a* 1*-sphere.*

Proof. The basic reference for the facts stated in the proof is [28].

Consider the following exact sequence [28, p. 453]:

$$1 \to Z_2 \to SO(4) \xrightarrow{\eta} SO(3) \times SO(3) \to 1.$$

Let I_{60} be a subgroup of $SO(3)$ isomorphic to the icosahedral group. Let $I_{120} = \eta^{-1}(I_{60} \times 1)$. Then $I_{120} \subset SO(4)$ is isomorphic to the binary icosahedral group and acts on S^3 fixed point freely by isometries. We shall take $p: S^3 \to D^3 = S^3/I_{120}$ as a standard universal covering of the Poincaré homology 3-sphere.

(i) We first prove the existence. Let $\{\tilde{f}''\} \subset SO(3)$ be a cyclic group of order n. Let $\tilde{f} \in \eta^{-1}(1 \times \tilde{f}'')$ and let $\widetilde{F}$ be the subgroup of $SO(4)$ generated by I_{120} and $\tilde{f}$. Note that $\tilde{f} I_{120} \tilde{f}^{-1} = I_{120}$, $\widetilde{F}$ is a group of isometries having I_{120} as a normal subgroup of index n and $\tilde{f}$ is a generator of the quotient group $\widetilde{F}/I_{120}$. Let I_{120} act on S^3 first and thus get the quotient space D^3. There is an induced orientation preserving isometric cyclic action on D^3 of order n as follows: let $p: S^3 \to D^3$ be the covering projection corresponding to the I_{120} action and define $f: D^3 \to D^3$ by $f(x) = p\tilde{f}(\tilde{x})$ where $x \in D^3$ and $\tilde{x} \in p^{-1}(x)$. Then f is well defined; in fact, let $\tilde{x}' \in p^{-1}(x)$, then there is $\alpha \in I_{120}$ such that $\alpha(\tilde{x}) = \tilde{x}'$ and thus $p\tilde{f}(\tilde{x}') = p\tilde{f}(\alpha(\tilde{x})) = p\beta\tilde{f}(\tilde{x}) = p\tilde{f}(\tilde{x})$ where $\beta = \tilde{f}\alpha\tilde{f}^{-1} \in I_{120}$. Similarly, using $\tilde{f}^{-1}$, define $f': D^3 \to D^3$ by $f'(x) = p\tilde{f}^{-1}(\tilde{x})$. It is easy to check that $f'f = 1$ and $ff' = 1$, and thus f is an isometry of D^3. As $fp = p\tilde{f}$, the order of f is n.

(ii) We now prove the uniqueness (up to conjugation by an isometry). Let $g: D^3 \to D^3$ be an orientation preserving isometry of order n. We may assume that the geometric structure on D^3 is induced from the universal covering

$p: S^3 \to D^3$ given at the beginning of the proof. We shall prove that, up to a conjugation by an isometry of D^3, the $\{g\}$ action is equivalent to the $\{f\}$ action given in (i).

Let $\widetilde{G} = \{\tilde{g}; \tilde{g}: S^3 \to S^3$ a map with $p\tilde{g} = g^k p$ for some integer $k\}$. Then $\widetilde{G} \subset SO(4)$ by our construction. By Lemma 2.1, $I_{120} \subset \widetilde{G}$ is a normal subgroup of index n. More explicitly, $\widetilde{G} = \bigcup_{k=1}^{k=n} \tilde{g}^k I_{120}$ for some $\tilde{g} \in \widetilde{G}$ with $p\tilde{g} = gp$.

Claim 1. There is an element $\tilde{h} \in SO(4)$ such that $\tilde{h}\widetilde{G}\tilde{h}^{-1} = \widetilde{F}$.

Proof of Claim 1. Still consider the exact sequence

$$1 \to Z_2 \to SO(4) \xrightarrow{\eta} SO(3) \times SO(3) \to 1.$$

Let p_i, $i = 1, 2$, be the natural projections from $SO(3) \times SO(3)$ to its left and right $SO(3)$ factors respectively. Then we must have $p_1\eta(\widetilde{G}) = I_{60}$ since $SO(3)$ has no finite group containing I_{60} as a proper subgroup. Let $\eta(\tilde{g}) = \tilde{g}' \times \tilde{g}'' \in SO(3) \times SO(3)$. Then $\tilde{g}'' \neq 1$ since otherwise the kernel of η would be larger than Z_2. As $\tilde{g}' \in I_{60}$, $(\tilde{g}'^{-1} \times 1)(\tilde{g}' \times \tilde{g}'') = 1 \times \tilde{g}'' \in \eta(\widetilde{G})$. Suppose $\tilde{g}''$ has order m in $SO(3)$. Then we see $\eta(\widetilde{G}) = I_{60} \times \{\tilde{g}''\}$ and thus $m = n$ by Lemma 2.1.

Since isomorphic subgroups of $SO(3)$ are conjugate, there are $\tilde{h}'' \in SO(3)$ such that $\tilde{h}''\{\tilde{g}''\}\tilde{h}''^{-1} = \{\tilde{f}''\}$. So $\eta(\widetilde{G})$ is conjugate to $I_{60} \times \{\tilde{f}''\}$ in $SO(3) \times SO(3)$ by the element $1 \times \tilde{h}''$. Let $\tilde{h} \in \eta^{-1}(1 \times \tilde{h}'')$. Then since the kernel of η is Z_2 which is contained in both $\widetilde{G}$ and $\widetilde{F}$, $\tilde{h}\widetilde{G}\tilde{h}^{-1} = \widetilde{F}$. Note that $\tilde{h}I_{120}\tilde{h}^{-1} = I_{120}$.

Claim 2. There is an isometry $h: D^3 \to D^3$ such that $h\{g\}h^{-1} = \{f\}$.

Proof of Claim 2. Define $h: D^3 \to D^3$ by $h(x) = p\tilde{h}(\tilde{x})$ where $\tilde{x} \in p^{-1}(x)$. Then h is well defined. In fact, let $\tilde{x}' \in p^{-1}(x)$, then there is $\alpha \in I_{120}$ such that $\alpha(\tilde{x}) = \tilde{x}'$ and thus $p\tilde{h}(\tilde{x}') = p\tilde{h}(\alpha(\tilde{x})) = p\beta\tilde{h}(\tilde{x}) = p\tilde{h}(\tilde{x})$ where $\beta = \tilde{h}\alpha\tilde{h}^{-1} \in I_{120}$. Similarly, using $\tilde{h}^{-1}$, define $h': D^3 \to D^3$ by $h'(x) = p\tilde{h}^{-1}(\tilde{x})$ where $\tilde{x} \in p^{-1}(x)$. It is easy to check $h'h = 1$ and $hh' = 1$, and thus h is an isometry and $h^{-1} = h'$.

Now let $x \in D^3$,

$$\begin{aligned} hgh^{-1}(x) &= hgp\tilde{h}^{-1}(\tilde{x}) = hp\tilde{g}\tilde{h}^{-1}(\tilde{x}) = p\tilde{h}\tilde{g}\tilde{h}^{-1}(\tilde{x}) \\ &= p\tilde{f}_*(\tilde{x}) = f^k p(\tilde{x}) = f^k(x) \end{aligned}$$

where $\tilde{f}_* = \tilde{h}\tilde{g}\tilde{h}^{-1} \in \widetilde{F}$ has order n. Hence f^k has order n and thus $h\{g\}h^{-1} = \{f\}$.

(iii) Note that $SO(3) \times SO(3)$ is the orientation preserving isometry group of $SO(3)$ and the diagram

$$\begin{array}{ccc} SO(4) \times S^3 & \xrightarrow{\eta \times q} & SO(3) \times SO(3) \times SO(3) \\ \downarrow & & \downarrow \\ S^3 & \xrightarrow{q} & SO(3) \end{array}$$

commutes, where the two vertical arrows denote the actions on S^3 and $SO(3)$ respectively and q is the quotient map defined by the standard Z_2 action on S^3.

Note that an element $\tilde{g}' \times \tilde{g}'' \in SO(3) \times SO(3)$ acts on $SO(3)$ fixed point freely iff $\tilde{g}'$ is not conjugate to $\tilde{g}''$ in $SO(3)$. Also note that two elements in $SO(3)$ with finite orders can possibly be conjugate only when they have the same order. Hence if n is relative prime to 2, 3, 5, then any element in $I_{60} \times \{\tilde{g}''\}$ acts on $SO(3)$ freely since orders of elements in I_{60} can only be 2, 3, and 5. Hence we have a free induced Z_n action on D^3.

If n is not relative prime to 2, 3, or 5, then exactly those elements $\tilde{c}' \times \tilde{c}'' \in I_{60} \times \{\tilde{g}''\}$ with $\tilde{c}'$ and $\tilde{c}''$ having the orders 2,3, or 5 and being conjugate to each other have fixed point sets in $SO(3)$. Such elements exist. Hence in these cases, we obtain $I_{60} \times Z_k$, $k = 2, 3$, or 5, actions on $SO(3)$ with fixed point sets. This in turn induces orientation preserving Z_k actions on D^3 with fixed point sets. By Smith theory [6] the fixed point set of each such cyclic action is a 1-sphere in D^3. □

Corollary 2.3. *Let M be a closed irreducible 3-manifold with $\pi_1(M) = I_{120}$ and let $f: M^3 \to M^3$ be a homeomorphism of order n. If the fixed point set of f has dimension 1, then M is homeomorphic to the Poincaré homology sphere D^3. Furthermore the action is unique up to a conjugation by a homeomorphism of D^3 and the order n must be 2 or 3 or 5. Thus the associated branched covering is equivalent to one of the known cases given in* §1(iii).

The proof of Corollary 2.3 is based on the following result of Thurston.

Theorem 2.4 (W. Thurston). *Let M be an irreducible closed 3-manifold which admits a finite cyclic group action with fixed point set of dimension 1. Then M has a geometric decomposition. Furthermore if M is also atoroidal, then M admits a geometric structure such that the group action is by isometries.*

Proof of Corollary 2.3. Since $\pi_1(M) = I_{120}$, M is atoroidal by Dehn's lemma. By Theorem 2.4, M is homeomorphic to D^3 and f is an isometry. Note that f is necessarily orientation preserving since it has 1-dimensional fixed point set. Now apply Theorem 2.2. □

3. Knots having property I or $\hat{\text{I}}$

First we apply Casson's theorem (see [1]) to give a simple but effective criterion for knots in S^3 to have property $\hat{\text{I}}$ (Lemma 3.2).

Theorem 3.1 (A. Casson). *Let λ denote the Casson invariant of an oriented homology 3-sphere M; then*

(1) $\lambda(-M) = -\lambda(M)$, *where $-M$ denotes opposite orientation of M.*

(2) $\lambda(M) = 0$ *if* $\pi_1(M) = 1$.

(3) $\lambda(M) \equiv \mu(M) \bmod 2$, *where $\mu(M)$ is the Rohlin invariant of M.*

(4) *Let K be a knot in S^3 and let $S^3(K, 1/l)$ be the homology 3-sphere obtained from $1/l$-surgery on K. Let $\Delta_K(t)$ be the normalized Alexander polynomial of K, i.e., $\Delta_K(1) = 1$ and $\Delta_K(t^{-1}) = \Delta_K(t)$. Then*

$$\lambda(S^3(K, 1/l)) = l(1/2)\Delta_K''(1)$$

where $\Delta_K''(1)$ is the second derivative of $\Delta_K(t)$ valued at 1.

For a knot K in S^3, $\lambda(K) = (1/2)\Delta_K''(1)$ is called the Casson invariant of K. Note that $\lambda(K)$ is always an integer.

Let T denote the right-hand trefoil knot in S^3 and let D^3 denote the Poincaré homology 3-sphere. Since D^3 can be obtained by 1-surgery along T and $\Delta_T(t) = -1 + t + t^{-1}$, we have $\lambda(D^3) = (1/2)\Delta_T''(1) = 1$. Now suppose that $S^3(K, 1/l)$, a manifold obtained by $1/l$-surgery along a knot K in S^3, is homeomorphic to D^3. Then by Theorem 3.1, $\lambda(S^3(K, 1/l)) = l(1/2)\Delta_K''(1) = 1$ or -1. Therefore $l = 1$ or -1 and $(1/2)\Delta_K''(1) = 1$ or -1. This gives

Lemma 3.2. *Let K be a knot in S^3. If $S^3(K, 1/l)$ is the Poincaré homology 3-sphere, then $\lambda(S^3(K, 1/l)) = 1$ or -1, $l = 1$ or -1, and $\lambda'(K) = (1/2)\Delta_K''(1) = 1$ or -1.*

Lemma 3.3. *If the Arf invariant, $\alpha(K)$, of a knot K in S^3 is trivial, then K has property $\hat{\mathrm{I}}$.*

Proof. In [10] González-Acuña established a surgery formula for calculating the Rohlin invariant of a homology 3-sphere $S^3(K, 1/l)$, in terms of surgery slope and the Arf invariant, that is,

$$\mu(S^3(K, 1/l)) \equiv l\alpha(K) \pmod 2.$$

By Theorem 3.1, $\lambda(S^3(K, 1/l)) \equiv \mu(S^3(K, 1/l)) \equiv l\alpha(K) \equiv 0 \pmod 2$. Hence $S^3(K, 1/l)$ cannot be the Poincaré homology 3-sphere by Lemma 3.2. □

Similarly one can show that a knot K in S^3 with Arf invariant 1 has property P. Also note $\lambda(K) \equiv \alpha(K) \pmod 2$.

Proposition 3.4. *Slice knots (and hence ribbon knots) have property $\hat{\mathrm{I}}$.*

Proof. It is known that the Arf invariant is an invariant of concordance [26]. Since any slice knot is concordant with the trivial knot and the Arf invariant of the trivial knot is 0, the proposition follows from Lemma 3.3. □

Proposition 3.5. *If two knots K_1 and K_2 in S^3 have the same Arf invariant, then any band-connect sum, $K_1 \#_b K_2$, of K_1 and K_2 has property $\hat{\mathrm{I}}$.*

Proof. In [16], Kauffman defined a Γ-equivalence relation for knots in S^3 and showed that two knots in S^3 are Γ-equivalent if and only if they have the same Arf invariant. Performing Γ-moves, one can easily show that $K_1 \#_b K_2$ is Γ-equivalent to $K_1 \# K_2$, the composite knot of K_1 and K_2. Hence they have the same Arf invariants. Since the Arf invariant is additive with respect to the knot connect sum, the proposition follows by Lemma 3.3. □

Similarly one can prove that if two knots K_1 and K_2 in S^3 have different Arf invariants, then $K_1 \#_b K_2$ has property P. Note that property $\hat{\mathrm{P}}$ for nontrivial band-connected sums was proved by Thompson [31] using a different method.

Proposition 3.6. *Nontrefoil torus knots have property I.*

Proof. This proposition is implicitly contained in [23]. Here we give a proof using the Casson invariant. Let $T(p, q)$ be a torus knot. Note that $(p, q) = 1$. We may also assume that $0 < p < q$. If $p = 1$, then $T(1, q)$ is the trivial knot which obviously has property I. So we may assume that $1 < p < q$. Note also

that $T(2, 3)$ is the trefoil knot and hence to be nontrefoil, $p \neq 2$ or $q \neq 3$. It is known that the normalized Alexander polynomial of $T(p, q)$ is

$$\Delta_{T(p,q)}(t) = t^{-(p-1)(q-1)/2}\frac{(1-t)(1-t^{pq})}{(1-t^p)(1-t^q)}.$$

Pure calculation of the second derivative of $\Delta_{T(p,q)}(t)$ gives $(1/2)\Delta''_{T(p,q)}(1) = (p^2-1)(q^2-1)/24$. Since $1 < p < q$ and $p \neq 2$ or $q \neq 3$, $(1/2)\Delta''_{T(p,q)}(1) > (3^2-1)(2^2-1)/24 = 1$. By Lemma 3.2, $T(p, q)$ has property $\hat{\mathrm{I}}$. Note that every manifold $S^3(T(p, q), 1/l)$ is Seifert fibered, so it cannot be a fake Poincaré homology 3-sphere. Hence $T(p, q)$ has property I. □

Similarly one can show that nontrivial torus knots have property P (the result was first proved by Hempel [14]). Therefore nontrivial nontrefoil torus knots have property PI.

Proposition 3.7. *Satellite knots have property* I.

Property P for satellite knots has been proved by Gabai [8]. His proof is based on the following result of his.

Theorem 3.8 [8]. *Let K be a knot in a solid torus N with nonzero wrapping number. Let $N(K, m/l)$ be the manifold obtained by performing m/l-surgery along K in N. Then one of the following must hold*:

(1) *$N(K, m/l)$ is a solid torus and K is a* 0- *or* 1*-bridge braid in N.*
(2) *$N(K, m/l) = Y \# W$, where W is a closed* 3*-manifold and $H_1(W)$ is finite and nontrivial.*
(3) *$N(K, m/l)$ is irreducible and $\partial N(K, m/l)$ is incompressible.*

To prove Proposition 3.7 we need another result of Gabai and a result of Gordon.

Theorem 3.9 [9]. *Let K be a knot in a solid torus N. If K is a* 1*-bridge braid, then only the surgery with slope $\pm(t + j\omega)\omega \pm b$ or $\pm(t + j\omega)\omega \pm b \pm 1$ on K can possibly yield a solid torus, where ω is the winding number of K in the solid torus, $t + j\omega$ is the twist number of K with $0 < t < \omega - 1$ (j being an integer), b is the bridge width of K with $0 < b < \omega - 1$.*

See [9] for the definitions of twist number and bridge width of a 1-bridge braid in a solid torus.

Lemma 3.10 [11]. *Let $K = C(p, q)$ be a cabled knot in a solid torus N. Then $N(K, m/l)$ is a solid torus iff $m = lpq \pm 1$.*

Proof of Proposition 3.7. Let K be a satellite knot in S^3 with K_* as a nontrivial companion knot. Let V and N be tubular neighborhoods of K and K_* in S^3 with $V \subset \operatorname{int} N$. Let $E = S^3 - \operatorname{int} V$, $E_* = S^3 - \operatorname{int} N$, and $E_0 = N - \operatorname{int} V$. Then $E = E_* \cup E_0$. Let μ, $\lambda \subset \partial E$ and μ^*, $\lambda^* \subset \partial E_*$ be preferred meridian-longitude pairs of K and K_* respectively. Let ω be the winding number of K in N.

Suppose that $S^3(K, 1/l)$ is a manifold with fundamental group I_{120}. Then ∂N must be compressible in $S^3(K, 1/l)$ by Dehn's lemma. Let $(D^2, \partial D^2) \subset (S^3(K, 1/l), \partial N)$ be a compressing 2-disc. Since ∂N is incompressible in E_*,

$(D^2, \partial D^2) \subset (N(K, 1/l), \partial N)$. Hence consider the manifold $N(K, 1/l)$, case (3) of Theorem 3.8 cannot happen. Case (2) of Theorem 3.8 cannot occur either by our assumption. Therefore $N(K, 1/l)$ is a solid torus and K is a 0- or 1-bridge braid in N. But by Theorem 3.9, K cannot be a 1-bridge braid and by Lemma 3.10, K cannot be a 0-bridge braid. A contradiction is thus obtained. □

Hence satellite knots have property PI. By Propositions 3.6 and 3.7, one only needs to show property I for hyperbolic knots.

Recall that a generalized doubled knot is defined as follows. Let V be an unknotted solid torus and let $K_{p,0}$ be the knot contained in V as shown in Figure 3.1(a). Let K_* be any knot in S^3 and let N be a tubular neighborhood of K_* in S^3. Let f be a homeomorphism from V to N. Then the image $K = f(K_{p,0})$ of $K_{p,0}$ under f is called a *generalized doubled knot* and K_* is called a *companion knot* of $K = f(K_{p,0})$. Note when $p = 1$ this is just the usual definition of a doubled knot.

Proposition 3.11. *Nontrefoil generalized doubled knots have property* $\hat{I}$.

Proof. Let K be a generalized doubled knot in S^3 and let K_* be its companion knot. If K_* is a nontrivial knot, then K is a satellite knot and Proposition 3.7 applies. If K^* is the trivial knot, then K is a generalized twisted knot (Figure 3.1(b)). So we assume that $K = K_{p,q}$, a generalized twisted knot with q twists. Note that $K_{p,0}$ is the trivial knot, $K_{1,-1}$ is the right-hand trefoil knot, $K_{-1,1}$ is the left-hand trefoil knot, $K_{-1,-1}$ and $K_{1,1}$ are the figure eight knot, and $K_{0,q}$ is the trivial knot.

Using the Conway recursion formula [7], one can easily obtain that the normalized Alexander polynomial of $K_{p,q}$ is $\Delta_{K_{p,q}}(t) = 2pq + 1 - pq(t + t^{-1})$. So $(1/2)\Delta''_{K_{p,q}}(1) = pq$. Hence by Lemma 3.2 only when $p = \pm 1$ and $q = \pm 1$ could $K_{p,q}$ have chance to ruin property $\hat{\mathrm{I}}$. But then $K_{p,q}$ is either a trefoil knot or a figure eight knot. It is well known that 1 and -1 surgeries on the figure eight knot produce the same manifold (the figure eight knot is amphicheiral) whose fundamental group is the triangle group with presentation $\{x, y; x^2 = y^3 = (xy)^7\}$ and thus is of infinite order. Therefore the figure eight knot has property I. This completes the proof. □

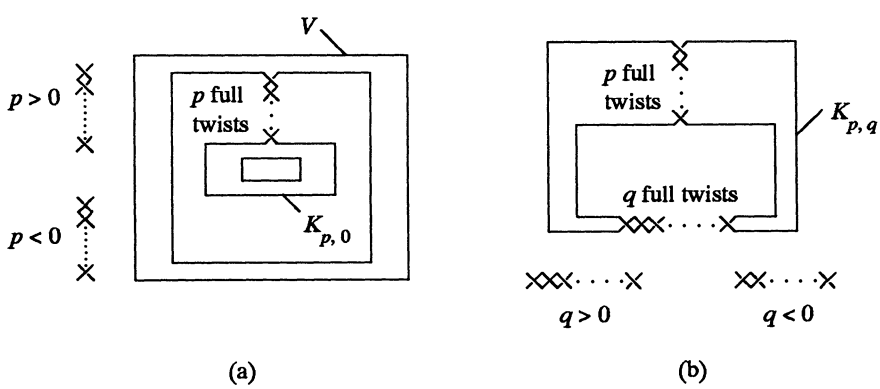


FIGURE 3.1.

Similarly one can show that nontrivial generalized double knots have property P.

Recall that a knot K in S^3 is called a *periodic* knot if there is an orientation preserving automorphism f of S^3 with the following properties: (i) f has period $n > 1$, that is, f^n is the identity map and f^i is not the identity map for $1 \leq i < n$; (ii) K is invariant under f, that is, $f(K) = K$; (iii) the fixed points set of f is not empty and is disjoint from K. Note that the action on S^3 by the cyclic transformation group $\{f\}$ generated by f induces a n-fold cyclic branched covering $p: S^3 \to S^3/\{f\}$. Due to the positive answer to the Smith conjecture [2], the map f is a rotation of S^3, $S^3/\{f\}$ is homeomorphic to S^3, the fixed point set of f is a trivial knot in S^3; and the image of the fixed point set under p is also a trivial knot in S^3. The restriction of p on K gives a regular covering $p: K \to p(K)$ and thus $p(K)$ is also a knot in $p(S^3) = S^3$. $p(K)$ is called a *factor* knot of K. The following lemma may be found in the literature so its proof is omitted here.

Lemma 3.12. *Let K be a periodic knot in S^3 with period n. If $(m, nl) = 1$, then $S^3(K, m/l)$ admits a Z_n action with fixed point set a 1-sphere. The quotient space of $S^3(K, m/l)$ under the action is $S^3(p(K), m/nl)$.*

Proposition 3.13. *Surgery on a periodic knot K in S^3 cannot give a fake Poincaré homology 3-sphere. A periodic knot in S^3 with period $n \neq 2, 3, 5$ has property* I.

Proof. By Lemma 3.12, $S^3(K, 1/l)$ admits a Z_n action with fixed point set a 1-sphere. By [12], any homology 3-sphere obtained by surgery on a knot in S^3 is irreducible. If for some slope $1/l$, $S^3(K, 1/l)$ has fundamental group I_{120}, then Corollary 2.3 implies that $S^3(K, 1/l)$ is the honest Poincaré homology 3-sphere. Also if $n \neq 2, 3, 5$, then $S^3(K, 1/l)$ cannot be the Poincaré sphere. □

Proposition 3.14. *A periodic knot K with a nontrivial factor knot has property* I.

Proof. By Proposition 3.13 and Lemma 3.2, we only need to show $S^3(K, \pm 1)$ is not homeomorphic to the Poincaré homology 3-sphere. By Lemma 3.12, $S^3(K, \pm 1)$ is the n-fold cyclic branched cover of $S^3(p(K), \pm 1/n)$ with branch set a 1-sphere. Since $S^3(p(K), \pm 1/n)$ cannot be the 3-sphere S^3 by [12], $S^3(K, \pm 1)$ cannot be the Poincaré homology 3-sphere by Corollary 2.3. □

Example 3.15. The knot 8_{18} is a periodic knot of period 2 with the figure eight knot 4_1 as a factor knot, and thus has property I.

Example 3.16. [1]

Recall that a knot K in S^3 is *strongly invertible* if there is an orientation preserving involution of S^3 which carries K onto itself and reverses its orientation. Note that the axis of the involution meets K in exactly two points.

The facts in this paragraph are found in [22, 3]. Let K be a strongly invertible knot in S^3. Then the restriction of the involution to the knot complement

[1] This example was deleted by the author after the paper was in proof. Please disregard any reference to it in text.

can be extended to an involution of the manifold $S^3(K, m/l)$ obtained by performing m/l-surgery on K. The quotient space of $S^3(K, m/l)$ under this involution is the 3-sphere S^3, i.e., $S^3(K, m/l)$ is a double branched cover of S^3. Moreover the branched set downstairs of this covering can be obtained by removing a trivial tangle from the unknot (the branched set corresponding to the trivial surgery) and replacing it by the m/l-rational tangle. In particular if the surgery slope is an integer m, then the removal and replacement of the trivial tangle corresponding to the surgery is in fact the attachment of a band with m half-twists to the unknot.

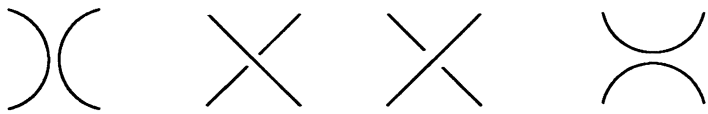
By the above discussion, $S^3(K, 1/l)$ admits a Z_2 action with fixed point set a 1-sphere. Hence by the same reason as given in the proof of Proposition 3.13, we have

Proposition 3.17. *Surgery on a strongly invertible knot K cannot yield a fake Poincaré homology 3-sphere.*

Proposition 3.18. *At most one surgery on a strongly invertible knot K can give a manifold with fundamental group I_{120}.*

Proof. By Proposition 3.17 and Lemma 3.2, we only need to show that $S^3(K, 1)$ and $S^3(K, -1)$ cannot both be homeomorphic to the Poincaré homology 3-sphere. Suppose, on the contrary, that they both are homeomorphic to the Poincaré homology 3-sphere. By Corollary 2.3, there is, up to a conjugation by a homeomorphism, a unique involution on the Poincaré homology 3-sphere with fixed point set a 1-sphere. Hence the associated double branched covering is the one mentioned in §1(iii). The branched set in the base space S^3 is the $(3, 5)$ torus knot up to unoriented automorphisms of S^3 and thus is either the right-hand or the left-hand $(3, 5)$ torus knot.

The branched sets corresponding to $S^3(K, 1)$ and $S^3(K, -1)$, denoted by K_1 and K_{-1}, can be obtained by band attachments with 1 and -1 half twist to the unknot respectively. Let U denote the unknot and let L_0 denote the link (of two components) obtained by band attachment with no twist to the unknot. Then K_1, K_{-1}, U and L_0 have diagrams differing only at the site shown below.



U K_1 K_{-1} L_0

We can orient K_1, K_{-1} and L_0 in a consistent way such that we can apply the Conway recursion formula and get $\nabla_{K_1} - \nabla_{K_{-1}} - z\nabla_{L_0} = 0$, where ∇ is the Conway polynomial. Since each of K_1 and K_{-1} is the right-hand or left-hand $(3, 5)$ torus knot, it is easy to show that $\mathrm{lk}(L_0) = 0$ (where lk denotes the linking number), using the following properties of the Conway polynomial: (1) if K is a knot in S^3, then ∇_K is independent of the choice of orientation for K; (2) let L be a link in S^3 and let L^* denote the mirror image of L, then $\nabla_{L^*}(z) = \nabla_L(-z)$; (3) let $\nabla_L(z) = a_0 + a_1 z + \cdots + a_n z^n$ be the Conway polynomial of a link L, then

$$a_1 = \begin{cases} \mathrm{lk}(L) & \text{if } L \text{ has two components,} \\ 0 & \text{otherwise.} \end{cases}$$

Now we try to get a contradiction by calculating the Kauffman brackets of K_1, K_{-1}, U, and L_0. Recall that the Kauffman bracket $\langle L\rangle(A) \in Z[A, A^{-1}]$ is defined for unoriented link diagrams L with the following defining relations:

(1) $$\langle \times\rangle = A\langle \asymp\rangle + A^{-1}\langle)(\rangle, \quad \langle \times\rangle = A^{-1}\langle \asymp\rangle + A\langle)(\rangle,$$

where $\times, \times, \asymp,)($ stand for link diagrams which look like that in a neighborhood of a point and identical elsewhere.

(2) $$\langle O\rangle = 1, \qquad \langle O \cup L\rangle = (-A^2 - A^{-2})\langle L\rangle,$$

where O denotes the unknot diagram with no crossing points and $\cup$ denotes the disjoint union.

$\langle L\rangle(A)$ is not a link invariant but it can be adjusted to be one for oriented links. Given an oriented link diagram L. Let $w(L)$ be the algebraic sum of the crossings of L, counting $\overrightarrow{\times}$ and $\overleftarrow{\times}$ as $+1$ and -1 respectively. Then

$$f_L(A) = (-A)^{-3w(L)}\langle L\rangle(A)$$

is a desired invariant of oriented links under ambient isotopy. We shall call $f_L(A)$ the *oriented* Kauffman bracket of L. Note that (1) if L is a knot then $f_L(A)$ is independent of the choice of orientation; (2) let L^* denote the mirror image of L, then $f_{L^*}(A) = f_L(A^{-1})$; (3) $f_L(t^{-1/4})$ is the Jones polynomial. For more details see [17].

For unoriented K_1, K_{-1}, U, and L_0, we have

(*) $$\langle K_1\rangle = A\langle L_0\rangle + A^{-1}\langle U\rangle, \quad \langle K_{-1}\rangle = A^{-1}\langle L_0\rangle + A\langle U\rangle.$$

Now consider the oriented K_1, K_{-1}, L_0, and U (the first three have consistent orientations and the orientation of U is arbitrarily given). Let $w(L_0) = n$. Then $w(U) = n$ since $\mathrm{lk}(L_0) = 0$. Also $w(K_1) = n + 1$ and $w(K_{-1}) = n - 1$. Hence $f_{L_0}(A) = (-A)^{-3n}\langle L_0\rangle$, $f_U(A) = (-A)^{-3n}\langle U\rangle = 1$, $f_{K_1}(A) = (-A)^{-3(n+1)}\langle K_1\rangle$, and $f_{K_{-1}}(A) = (-A)^{-3(n-1)}\langle K_{-1}\rangle$. Substituting them into (*) above, we have

(**) $$-A^2 f_{K_1}(A) = f_{L_0}(A) + A^{-2}, \quad -A^{-2} f_{K_{-1}}(A) = f_{L_0}(A) + A^2.$$

Eliminating f_{L_0}, we get $A^2 f_{K_1}(A) - A^{-2} f_{K_{-1}}(A) = A^2 - A^{-2}$. Hence we have either

(i) $f_{K_1} = f_{K_{-1}} = 1$ if K_1 is ambient isotopic to K_{-1}; or

(ii) $A^2 f_{K_1}(A) - A^{-2} f_{K_1}(A^{-1}) = A^2 - A^{-2}$ if K_1 is the mirror image of K_{-1}.

But the oriented Kauffman brackets of right-hand and left-hand $(3, 5)$ torus knots are $f(A) = A^{-16} + A^{-24} - A^{-40}$ and $f(A^{-1})$ neither of which fit (i) or (ii). □

For an amphicheiral knot K in S^3, $S^3(K, m/l) = S^3(K, -m/l)$. Hence we have

Corollary 3.19. *Amphicheiral strongly invertible knots have property* I.

Bleiler and Scharlemann have shown property P for strongly invertible knots [5]. Hence amphicheiral strongly invertible knots have property PI.

Example 3.20. The knot 6_3 is an amphicheiral strongly invertible knot and hence has property I.

Proposition 3.21. *Let K be a pretzel knot of type (p, q, r) such that r is an even number, $p+q \neq 0$, p, q are not relative prime. Then K has property I.*

Proof. Since these pretzel knots are strongly invertible, we only need worry about ± 1 surgeries by Proposition 3.17 and Lemma 3.2. A method used by Simon [29] in proving property P for these knots can be easily generalized to work for property I and we omit the details here. □

Example 3.22. The knot 8_5 is a pretzel knot of type $(3, 3, 2)$ and thus has property PI by Proposition 3.21.

Note that Ortmeyer showed in [25] that R^3 is the universal cover of each manifold obtained by nontrivial surgery on pretzel knot of type $(4+2p, 3+2q, -5-2r)$ with p, q, r positive. Hence this family of pretzel knots have property PI.

Computing $\lambda(K) = 1/2\Delta_K''(1)$ for the classical knots up to nine crossings, we obtain the following table of their Casson invariants (we use the knot table given in [27]).

knot	3_1	4_1	5_1	5_2	6_1	6_2	6_3	7_1	7_2	7_3	7_4	7_5	7_6	7_7
$\lambda(K)$	1	−1	3	−2	−2	1	1	6	3	5	4	4	1	1
knot	8_1	8_2	8_3	8_4	8_5	8_6	8_7	8_8	8_9	8_{10}	8_{11}	8_{12}	8_{13}	8_{14}
$\lambda(K)$	−3	0	−4	−3	−1	−2	2	2	−2	3	−1	−3	1	0
knot	8_{15}	8_{16}	8_{17}	8_{18}	8_{19}	8_{20}	8_{21}	9_1	9_2	9_3	9_4	9_5	9_6	9_7
$\lambda(K)$	4	1	1	1	5	2	0	10	4	9	7	6	7	5
knot	9_8	9_9	9_{10}	9_{11}	9_{12}	9_{13}	9_{14}	9_{15}	9_{16}	9_{17}	9_{18}	9_{19}	9_{20}	9_{21}
$\lambda(K)$	0	8	8	4	1	7	−1	2	6	−2	6	−2	2	3
knot	9_{22}	9_{23}	9_{24}	9_{25}	9_{26}	9_{27}	9_{28}	9_{29}	9_{30}	9_{31}	9_{32}	9_{33}	9_{34}	9_{35}
$\lambda(K)$	−1	5	1	0	0	0	−1	−1	−1	2	1	1	−1	7
knot	9_{36}	9_{37}	9_{38}	9_{39}	9_{40}	9_{41}	9_{42}	9_{43}	9_{44}	9_{45}	9_{46}	9_{47}	9_{48}	9_{49}
$\lambda(K)$	3	−3	6	2	−1	0	−2	−1	0	2	−2	−1	3	6

This calculation gives immediately that 59 out of the 84 knots have property $\hat{\mathrm{I}}$ ($\lambda(K) \neq \pm 1$). But these 59 knots are strongly invertible [13], hence they have property I. Property I for the knots $4_1, 6_3, 8_5, 8_{18}$ has been shown in this section. Except for $8_{17}, 9_{32}$, and 9_{33}, the rest of the knots are strongly invertible [13] and one thus could decide property I for these knots, namely, consider the double branched cover associated to the 1 or -1 surgery, find the branched knot in the base S^3 and check if it is a torus knot of type $(3, 5)$ or its mirror image.

All nontrivial knots with nine or fewer crossings have property P since 75 of them have $\lambda(K) \neq 0$ and the rest are strongly invertible.

4. Further remarks and open problems

From the discussion in §3, we see that to solve property I for periodic knots, it is equivalent to solve the following

Question 4.1. Let K be a periodic knot with period 2 or 3 or 5 and with a trivial factor knot $p(K)$. When can the branched set (which is a trivial knot),

in S^3 downstairs of the covering $p: S^3 \to S^3/\{f\}$ become a torus knot of type $(\pm 3, 5)$ or $(\pm 2, 5)$ or $(\pm 2, 3)$ after performing $\pm 1/2$ or $\pm 1/3$ or $\pm 1/5$ surgery on $p(K)$ in S^3?

Also from §3 w see that to solve property I for strongly invertible knots in S^3, it is enough to solve the following

Question 4.2. When can a trivial knot be changed into a torus knot of type $(3, 5)$ or $(-3, 5)$ after attached a band with a half twist?

Let K be a knot in S^3 and let $E = S^3 - \operatorname{int} N(K)$ be its knot complement. Suppose F is a closed connected nonperipheral incompressible surface in E. Note that F is necessarily orientable and it separates E into two components, say E_1 and E_2, that is, $E = E_1 \cup E_2$, $E_1 \cap E_2 = \partial E_1 \cap \partial E_2 = F$. Assume that E_2 is the component which contains ∂E. The surface F is called an *m-surface* if there is an annulus A properly embedded in E_2 with ∂A consisting of a 1-sphere in F and a meridian curve in ∂E. F is called an *2m-surface* if there are two disjoint annuli A_1 and A_2 properly embedded in E_2 with $\partial A_1 = s_1 \cup m_1$ and $\partial A_2 = s_2 \cup m_2$ such that s_1 and s_2 are nonisotopic simple closed curves in F and that m_1 and m_2 are meridian curves in ∂E. In [21] Menasco proved that if K is a knot with a $2m$-surface F, then F remains incompressible in each manifold $S^3(K, m/l)$ obtained by a nontrivial surgery on K. Hence knots with $2m$-surfaces have property PI by Dehn's lemma. In [24] Oertel showed that a Montesinos knot of type $(p_1/q_1, \ldots, p_n/q_n)$ with $n \geq 4$, $q_i \geq 3$, $i = 1, \ldots, n$, is a knot with $2m$-surface. Therefore this family of Montesinos knots have property PI.

Question 4.3. Let $K \subset S^3$ be a knot with an m-surface F. Is it true that F remains incompressible in each manifold $S^3(K, m/l)$ with $m/l \neq 1/0$?

In [30] Takahashi proved that no nontrivial surgery on a nontorus 2-bridge knot K can produce a manifold with cyclic fundamental group. His idea is to show that corresponding to a nontrivial surgery on K there is a homomorphism from the fundamental group of the resulting manifold to the group $GL(2, C)$ with noncyclic image.

Question 4.4. For 1 or -1 surgery on a nontorus 2-bridge knot, is there a homomorphism from the fundamental group of the resulting manifold to the group $GL(2, C)$ with infinite image?

Of course the positive answer implies property I for nontorus 2-bridge knots (note that 2-bridge knots are strongly invertible).

If there are no fake Poincaré homology 3-spheres, then property I is identical with property $\hat{\mathrm{I}}$ and things become much simpler by Lemma 3.2. For fake Poincaré homology 3-spheres there is also a control on surgery slopes. Recently Bleiler and Hodgson have shown [4] that if a hyperbolic knot in S^3 admits two finite surgeries then the distance between the two slopes is less than 21. Hence if $1/l$ surgery on a hyperbolic knot produces a fake Poincaré homology 3-sphere, then $|l| < 21$. To further eliminate the possibilities of obtaining fake Poincaré homology 3-sphere by surgery on a knot in S^3, one approach one could consider is suggested by the following two questions.

Question 4.5. If $S^3(K, 1/l)$ is a fake Poincaré homology 3-sphere, is it homotopy equivalent to the honest Poincaré homology 3-sphere?

Question 4.6. Is the Casson invariant a homotopy type invariant?

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